UTILIZING NEWTON’S LAW OF COOLING TO CORROBORATE THE MPEMBA EFFECT

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1. INTRODUCTION

Newton presented the Law of Cooling in a paper entitled "Scala Graduum Caloris" in the Philosophical Transactions of the Royal Society of London, in 1701. In that time, there were not designated temperature scales, so Newton measured the expansion of linseed oil and used that as a reference temperature scale.

After making this thermometer, he decided to measure the rate at which iron cooled. Using this data, he was able to derive the Law of Cooling. His equation is still useful today, as predicting the time at which a substance will cool to a certain temperature can save time and money.

The Mpemba effect states that water freezes faster when it is heated first. I plan to perform a real world experiment to see if it that is the case, and to use Newton’s Law of Cooling as further evidence. I have not been able to find any similar experiments with the method that I have in mind.

2. METHODS

I. Deriving equations for the experiment:

If something is being heated up we can define \( \frac{dT}{dt} \) as proportional to the change in temperature \( (T_1 - T_2) \) where:
- \( \frac{dT}{dt} \) is the change in temperature \( T \) with respect to time \( t \)
- \( T_1 \) is the initial temperature
- \( T_2 \) is the ambient temperature

If \( T_1 - T_2 > 0 \) the substance is being heated, so in our case \( T_1 - T_2 < 0 \). This means we have a negative value. This is where the proportionality constant comes in.

Newton’s Law of Cooling is given by: \( \frac{dT}{dt} = -k(T_1 - T_2) \)

\( k \) is the proportionality constant and is unique for each substance and circumstance. In order to perform the experiment, we must first find the \( k \) value.

\[ dT = -k * (T_1 - T_2) \, dt \]

\[ \frac{dT}{T_1 - T_2} = -k \, dt \]

Integrate both sides to get rid of \( dT \) and \( dt \):

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\[
\int_{T(0) - T_s}^{T(t) - T_s} \frac{1}{T_1 - T_2} dT = \int_{0}^{t} -k \, dt
\]

Where \( T_s \) = Surrounding temperature & \( T_1 - T_2 = T \)

\[
\ln(T(t) - T_s) - \ln(T(0) - T_s) = -kt
\]

\[
\ln \left[ \frac{T(t) - T_s}{T(0) - T_s} \right] = -kt \quad \text{Solving for } k \quad k = \frac{\ln \left[ \frac{T(t) - T_s}{T(0) - T_s} \right]}{t}
\] Equation 1

In order to get a value for the proportionality constant (k) we must have boundary conditions. These will be obtained from experimental data. All we need is a temperature at a certain time \( T(t) \) and the beginning temperature \( T(0) \). After we find that value we can plug it in, along with the constant \( T_s \) (defined above), to obtain a number for k.

Note: Whenever I did the experiment, I found it best to take the average of the k values around the desired temperature of water (freezing), for the best approximation when using Newton’s Law in the form below.

\[
\frac{T(t) - T_s}{T(0) - T_s} = e^{-kt} \quad \text{Solving for } T(t) \quad T(t) = e^{-kt} \ast (T(0) - T_s) + T_s
\] Equation 2

We can use this equation to solve for temperature as a function of time.

II. Performing the experiment – ETC ≈ 3 – 4 hours

Material list:
1. 8 oz. (1 cup) measuring spoon
2. Water
3. 2 Plastic cups
4. 2 Glass cups
5. Freezer
6. Stove with a pot
7. Water-resistant temperature probe (Stem thermometer)
8. Infrared thermometer
9. Stopwatch
10. Computer with Excel

Procedure:
Part 1: Room Temperature
1. Start by setting all four cups out so that they all reach the same temperature. We are doing this to eliminate the variable of the starting temperature of the cup. The cups could be cooled to the temperature of the freezer, but performing Part 2 with a cooled glass cup would not be safe.

2. Record the ambient temperature of the freezer with and without the blower on. Take the average of these two temperatures (this will be the \( T_s \) value).

3. Use measuring spoon to pour 1 cup of water into 1 glass cup and 1 plastic each. Set the other two cups aside.

4. Record the temperature of the water in each cup with the probe and infrared thermometers (this will be the \( T(0) \) value).
5. Start the stopwatch and set both cups in the freezer at different times (noting the initial time). There should be a large enough time gap while the water is freezing so that once one is completely frozen, the other will still be liquid. I would recommend at least a 10 minute time gap.

6. Record the water temperatures in both cups with each thermometer at regular time intervals until the water in each cup freezes completely. When the water is near freezing for one cup, set the probe in that cup until the water is solid. DO NOT DISTURB THE CUP WHILE THIS PROCESS IS HAPPENING.

   Note: Whenever I performed the experiment my time intervals were somewhat variable, but I recorded the temperature of the water in each cup at the same time. I would recommend measuring the temperature every 5 or 10 minutes. Under my conditions, it took over an hour for the water to freeze.

7. After the water freezes and all data is taken, set the cups out to warm and then properly dispose of them.

**Part 2: Heated Temperature**

1. Repeat Part 1, leaving out step 1 and including the following:
2. Boil water in a pot on a stove between steps 2 and 3.
3. Alter Part 1 so that each cup will be done separately. This will allow for greater accuracy when measuring the initial temperature of the hot water in the cup.

**III. Data analysis and techniques**

Using the time and temperature values from the experiment, calculate the k values for each sample in Excel, using the proportionality constant equation given in Section I of this report. By making a graph of this data (see Fig. 3 from **Results**) a level plane should appear. Taking the average value of this plane will give the actual k value that will be used in Newton’s Law. This greatly reduces the error.

Now using this improved k value, *Equation 2* can be used to predict the temperature for any given time. I recommend making tables of time, predicted temperature, and real temperature for each cup, with each thermometer. This will show the error of the predicted values. Make two graph of the real vs. predicted with respect to time for room temperature and heated temperature (See Fig. 4 & 5 from **Results**).

**3. Results & Discussion**

Please contact Eric Henderson at ericthenderson@gmail.com or view file in the attached flash drive for complete experimental data. There was simply too much to include in this report. Instead of having pages and pages of data tables, I have chosen to include important graphs and variable values.
Figures 1 and 2 show temperature (T [°C]) vs. Time [min]. These graphs contain no predicted data. From these graphs we can see that the probe in the glass and plastic cups, and the infrared for the glass cup all followed a somewhat similar pattern for both of the cooling scenarios. The infrared for the plastic cup, on the other hand, had a skewed curve as it did not make accurate measurements in the plastic cup (source of error). These figures also corroborate the Mpemba effect, because they show that water will have a much greater change in temperature as it is cooling if it is heated first (Figure 1 is more linear, while Figure 2 is more exponential). This can be further evidenced by graphing the change in temperature (delta T) vs. the time.

Determining a good k value from the graphs is indicated by the circled areas. These data points were averaged to give a much better predicted temperature. For comparison, using the average of all the data points gave a predicted time of freezing that was 10 minutes off for the water in the glass cup using the probe, while using the improved k value cut down the error in time to about 4 minutes. The actual k values used:

\[ k \text{[min}^{-1}] \text{ from room temp vs. heated:} \]
- for water in glass cup using probe = 0.02189346 vs. 0.0522161
- for water in plastic cup using probe = 0.02597587 vs. 0.048267
- for water in glass cup using infrared= 0.020163 vs. 0.0430337
- for water in plastic cup using infrared = 0.0250152 vs. 0.0465611

Unfortunately, I was not able to find any book values for the proportionality constant of water for Newton’s Law of Cooling, so I am not able to compare my found values. However, my k values do show consistency.
Now this is where it gets really cool (pun intended).

Figure 4 shows a comparison of the predicted (from Newton’s Law) and real cooling data, while Figure 5 sums up this entire report in one graph. Figure 5 shows that Newton’s Law does indeed corroborate the Mpemba effect, but my real world experiment was not the case. Newton’s Law predicted the water in the glass to freeze at around 70 minutes for the non-heated water, and 60 minutes for the heated water. Eureka!

So why didn’t the water freeze faster when I performed the experiment? Newton’s Law of Cooling was within 10 minutes of freezing time for each non-heated sample, but only within 55 minutes for the heated samples. Why the large gap?

These factors could have come from a variety of errors that come with real world testing. Human error in timing, taking measurements, and making calculations, the type of material used to contain the water, an unstable ambient temperature, and equipment error. The error of measurement with the infrared thermometer is unknown, but the error with the probe is given on the back of the package and is: ±1 °C between -19.9 – 119.9 °C. Another factor that could have affected the freezing time of the water was the minerals in the water, since tap water was used.

Overall, I would say that my experiment was a success. Other scientific studies² have been done before to corroborate the Mpemba effect and their findings (not method) are very similar to this report. Although I didn’t prove the Mpemba effect in the real world, I was able to use my data along with Newton’s Law of Cooling to show that it is true. The amount of samples and data taken from my experiment, along with the logical accuracy of the data inspires confidence in my work.

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